FELIN 2023

Forward & inverse kinematics and DYNAMIC

# BODY REFERENCE FRAME (BRF)

* Right-hand coordinate system
* Centred on body geometric centre,
* X forward
* Y leftward
* Z upward

Une image contenant carte, texte, dessin au trait

Description générée automatiquement

In the default standing pose, the position of the feet in the body reference frame is defined by the following constants:

* Delta X (note: positive)
* Delta Y (note: positive)
* Shift X (note: a longitudinal displacement to centre the ground projection of the CoM between all feet)
* Default Z Reference (note: negative)

# Robot pose matrix

The pose of the robot is expressed using a 3x4 matrix.

Each column is the 3D coordinates of a foot.

The column order is:

In the default standing pose, the content of the pose matrix, in the body reference frame, is:

*Note: see Config.py in “default stance” member function.*

# LEG REFERENCE FRAME (LRF)

* Right-hand coordinate system
* Centred on hips revolute joints,
* X upward
* Y leftward
* Z backward

The leg reference frame is aligned with the first (abduction) hips revolute joint.

The origin of the legs is defined by the following constants:

* Leg Front-Back (note: positive)
* Leg Left-Right (note: positive)

The origin of a leg is **fixed** in the robot reference frame and located at the **intersection** of hips abduction and flexion/extension axis.

The 3D coordinates of the leg origins may be expressed by a 3x4 matrix. Each column is the 3D coordinates of a leg origin. The column order is the same as the robot pose matrix:

In the body reference frame, the content of the leg origin matrix is:

*Note: see Config.py in “leg origin” matrix.*

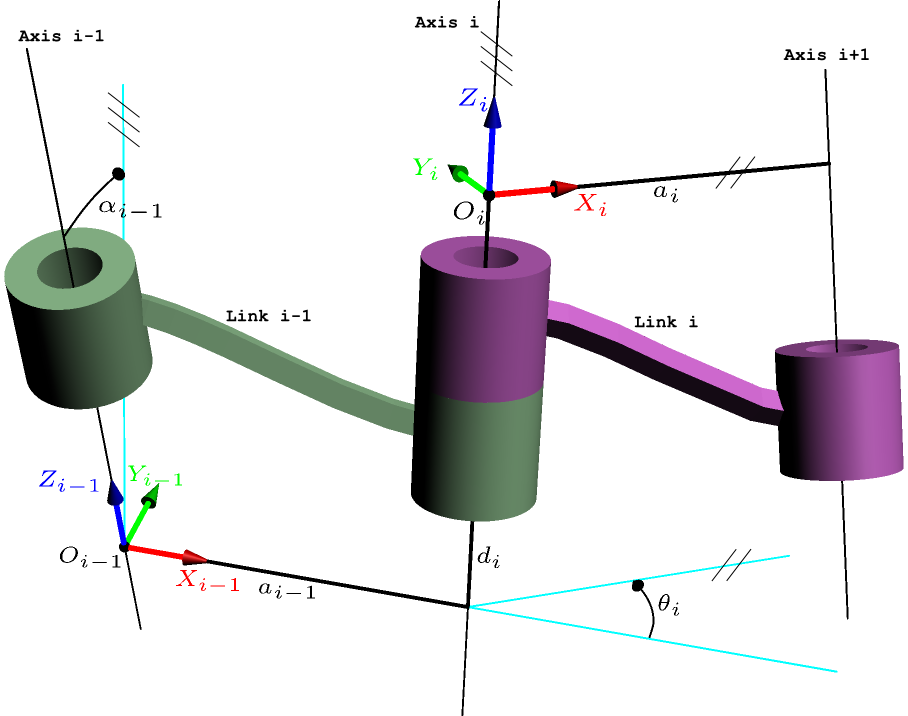
For the FR leg, the transformation matrix from LRF to BRF is:

For the FR leg, the transformation matrix from BRF to LRF is:

*Note: see Kinematics.py in “four legs inverse kinematics” function.*

# Forward kinematics using Modified Denavit-Hartenberg method (DHm)

## Modified DH Parameters



* is rotation about Z axis of this joint,
* **d** is translation along Z axis of this joint,
* **a** is length of link attached to this joint,
* **α** is twist of the link attached to this joint.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Links |  | d |  | a |
| 1 (hips/abduction) |  | 0 | -90° | 0 |
| 2 (hips flexion/extension) | ° | \* | 0° |  |
| 3 (knee) |  | 0 | 0° |  |

Note (\*): + for left legs | - for right legs.

where

|  |  |  |
| --- | --- | --- |
| Variables | Description | Values |
|  | Abduction offset | 60 mm |
|  | Length of femur | 140 mm |
|  | Length of tibia to centre of foot (foot radius = 10mm) | 150 mm |
|  | Hips (abduction) revolute joint angle, ZERO when VERTICAL | *controlled by servo #1* |
|  | Hips (flexion/extension) revolute joint angle, 180° when femur is HOR | *controlled by servo #2* |
|  | Knee revolute joint angle, 90° when TIBIA is VERTICAL | *controlled by servo #3* |

## Transformation matrices using modified DH parameters

### (reminder) modified DH transformation matrix

Transformation from coordinate frame j to coordinate frame j-1:

### HIPS (Abduction) (j=1)

where

|  |  |
| --- | --- |
|  |  |
|  |  |

### HIPS (FLEXION/EXTENSION) (j=2)

* For left legs:
* For right legs:

where

|  |  |
| --- | --- |
|  |  |
|  |  |

### KNEE (j=3)

where

|  |  |
| --- | --- |
|  |  |
|  |  |

### FOOT (j=4)

### « end-effector » Transformation matrix to leg reference frame

* For left legs:
* For right legs:

where

|  |  |
| --- | --- |
|  |  |
|  |  |

## Forward Kinematic

Position of foot in leg reference frame is given by the 3x1 submatrix of describing translation:

* For left legs:
* For right legs:

Where

|  |  |
| --- | --- |
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|  |  |
|  |  |
|  |  |

## Forward Kinematic (optimized)

Position of foot in leg reference frame is given by the 3x1 submatrix of describing translation:

* For left legs:
* For right legs:

Where

|  |  |
| --- | --- |
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# computation of jacobian

Velocity of the foot in leg reference frame #0 is given by the Jacobian  :

where

## Partial Differentiation Method

|  |  |
| --- | --- |
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|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

* For left legs:
* For right legs:

# computation of ground reaction force

Ground reaction force in leg reference frame #0 is given by the inverse transposed Jacobian and the torque of each joint:

where

## Singularities

Numerical approach may be used for singularities computation.

## INVERSE maTRIX (if NON-ZERO determinant)

Numerical approach may be used for inverse Jacobian matrix computation.

# Inverse KINEMATIC

Geometric approach.